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FOREIGN TECHNOLOGY DIVISION

PROBLEMS OF DIFFRACTION AND PROPAGATION OF WAVES

(Selected Articles)

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PROBLEMS OF DIFFRACTION AND PROPAGATION
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*<u>ye</u> initially, after vowels, and after ъ, ь; <u>e</u> elsewhere. When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

GRAPHICS DISCLAIMER

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THE TROPOSPHERIC REFRACTION OF RADIO WAVES.

G. I. Makarov, N. P. Tikhomirov.

Introduction.

This work is the continuation of our investigations [1, 2], which concerned direct and reverse/inverse problems of diffraction on the impedance sphere of a large radius. It is dedicated to the use/application of developed ones in [1, 2] methods for the solution of these problems taking into account the heterogeneous in the height/altitude atmosphere. For the investigation the exponential profile/airfoil, which is a most precise analytical approximation of the regular component of the atmospheric heterogeneity of the Earth [3, 5], is selected. A question about the role of tropospheric refraction was examined by many authors, in this case were studied different profiles/airfoils: bilinear [6], parabolic [7], exponential [8] and the profiles/airfoils of more general view, of which was required only sufficient "smoothness" [9-11].

The basic content of the mentioned works, with the exception of

[7], is confirmation of the validity of the concept of equivalent radius [9]. The conclusion of the author [7] the fact that for the parabolic profile/airfoil the concept of an equivalent radius is wrong, erroneous, as this was noted in [14].

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The most general/most common/most total analytical results, which relate to the consideration of tropospheric refraction, are contained in [11]. The author examined arbitrary smooth profile/airfoil, but were obtained for the eigenvalues hard-to-visualize expressions, the region of applicability of which was limited by high frequencies. A question about the role of tropospheric refraction at the low frequencies up to now remained uninvestigated. Furthermore, both at low and high frequencies it is important to have simple and sufficiently exact expressions for the eigenvalues. This would make it possible to come to light/detect/expose the possibility of the analytical solution of inverse problem. The study of these problems composes the content of this work.

§1. Formulation of the problem.

We will be interested in the distant field of vertical electric dipole on the impedance sphere of a large radius. Sphere we assume by

the surrounded inhomogeneous medium with the dielectric constant ϵ, which depends only on the radius: $\epsilon=\epsilon(r)$. Subsequently we will consider that the dependence $\epsilon(r)$ corresponds to the exponential profile/airfoil of atmospheric heterogeneity. Before extracting the formal solution of problem, which it is possible to construct, for example, by the method of normal waves, let us agree about the designations: spherical coordinates (r, θ, φ) is derived so that the surface of sphere coincides with the coordinate surface of r=a, angle ϑ is counted off from the polar axis, carried out through the radiating dipole (r=b, θ=0). Emitter is characterized by the current of amplitude I, which changes in the time according to harmonic law $e^{-i\omega t}$, and by effective height h_g. Let us extract now expression for the vertical component of electric field E_r in the form of the series/row of the normal waves

$$E_r = -\frac{i\omega\mu_0 I\cdot h_g}{4\sqrt{\varepsilon_m(b)\varepsilon_m(r)}}\cdot\frac{1}{k_0^2 b^2 r^2}\sum_{s=0}^{\infty}\frac{\nu_s(\nu_s+1)(2\nu_s+1)}{\frac{\partial}{\partial\nu}\left[\frac{\frac{\partial k_\nu}{\partial r}}{k_\nu}\right]_{\substack{r=a\\ \nu=\nu_s}}}\times$$

$$\times\frac{R_{\nu_s}(b)R_{\nu_s}(r)}{R^2_{\nu_s}(a)}\cdot\frac{P_{\nu_s}[\cos(\pi-\theta)]}{\sin\nu_s\pi}. \qquad (1)$$

Here $\varepsilon_m(r)=\frac{\epsilon(r)}{\epsilon_0}$ — relative dielectric constant of the atmosphere; $\epsilon_0=8.85\cdot10^{-12}$ F/m, $\mu_0=4\pi\cdot10^{-7}$ H/m; $k_0=\omega\sqrt{\epsilon_0\mu_0}$ - wave number in the vacuum, $P_{\nu_s}[\cos(\pi-\theta)]$ — Legendre's function: $R_{\nu_s}(r)$ — eigenfunctions; ν_s — the eigenvalues of radial operator L_r, which corresponds to the boundary-value problem

$$[L_r-\nu_s(\nu_s+1)]R_{\nu_s}(r)=0. \qquad (2)$$

1) $\left.\frac{dR_{\nu_s}}{dr}\right|_{r=a} = -ik_0\delta_g^* R_{\nu_s}(a)$, 2) $R_{\nu_s}(r) \in L^2[a, \infty)$ when $\operatorname{Im} \varepsilon_m > 0$, where $\delta_g^* \equiv \varepsilon_m(a)\delta_g - \frac{i}{2k_0\varepsilon_m(a)} \cdot \left.\frac{d\varepsilon_m}{dr}\right|_{r=a}$; δ_g — given surface impedance of sphere; $\varepsilon_m(a)$ — relative dielectric con... ant on its surface.

Differe: tial expression L_r takes the form

$$L_r \equiv r^2 \left\{ \frac{d^2}{dr^2} + k_0^2\varepsilon_m(r) - \sqrt{\varepsilon_m(r)} \frac{d^2}{dr^2} \frac{1}{\sqrt{\varepsilon_m(r)}} \right\} \quad (3)$$

Con... ering it carried out of the condition

$$|\nu_s| \gg 1, \ |\nu_s|(\pi - \theta) \gg 1, \ \operatorname{Im} \nu_s(\pi - \theta) > 1,$$

let us lead (1) to the standard form

$$E_r \simeq \frac{i\omega\mu_0 I \cdot h_g}{\ldots} \cdot \sqrt{\frac{2}{\pi k a \theta}} \sqrt{\frac{\theta}{\sin\theta}}\, e^{\ldots} \sum_{s=0}^{\ldots} \Lambda_s \cdot f_s(b) f_s(r)\, e^{i\nu_s\theta} \quad (4)$$

Are here introduced the following designations:

excitation coefficient s-th of the normal wave

$$\Lambda_s \equiv \frac{\nu_s^{1/2}(\nu_s+1)(2\nu_s+1)}{a(ka)^{3/2} \frac{\partial}{\partial\nu} \left[\frac{\frac{dR_\nu}{dr}}{R_\nu} \right]_{\substack{r=a \\ \nu=\nu_s}}};$$

the high-altitude factors

$$f_s(b) = \frac{\sqrt{\varepsilon_m(a)}}{\sqrt{\varepsilon_m(b)}} \cdot \frac{a^2}{b^2} \cdot \frac{R_{\nu_s}(b)}{R_{\nu_s}(a)},$$

$$f_s(r) = \frac{\sqrt{\varepsilon_m(a)}}{\sqrt{\varepsilon_m(r)}} \cdot \frac{a^2}{r^2} \cdot \frac{R_{\nu_s}(r)}{R_{\nu_s}(a)}.$$

The important characteristics of normal waves is their phase speed $v_{\text{фаз}}^{(s)}$ along the surface of sphere and attenuation factor $\alpha^{(s)}$.

For the s-th mode we have

$$v_{\text{фаз}}^{(s)} = \frac{ka}{\mathrm{Re}\,\nu_s}c, \qquad (5)$$

$$\gamma^{(s)} = \frac{\mathrm{Im}\,\nu_s}{a} \qquad (6)$$

It is known that the solution of transcendental equation relative to the marks of addition ν_s composes basic difficulty during further investigation of the series/row of normal waves (4) In the present work it is analogous how this was done in [1], we will attempt to obtain simple analytical dependences ν_s on impedance frequencies ω and the parameters of the heterogeneity of the δ_k atmosphere. In contrast to [1] we will be interested only in the first approximation of the Galerkin method.

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This method is applicable upon consideration of inhomogeneous medium, since under the impedance conditions on the surface of sphere the spectrum of operator L_r is discrete/digital. It is assumed that the spectrum of simple. For the realization of the Galerkin method more conveniently to switch over from L_r to integral operator which proves to be limited. Before carrying out this transition, let us rewrite (2), conditions 1) 2) in the new dimensionless variable

$$x = k(r - a).$$

where $k = k_0\sqrt{\varepsilon_m(0)}$, and $\varepsilon_m(0) = \varepsilon_m(x)|_{x=0}$ -- dielectric constant on the surface of the sphere:

$$\left\{\frac{d^2}{dx^2} + \frac{\varepsilon_m(x)}{\varepsilon_m(0)} - \sqrt{\varepsilon_m(x)}\frac{d^2}{dx^2}\frac{1}{\sqrt{\varepsilon_m(x)}} - \frac{\nu_s(\nu_s+1)}{(ka)^2}\cdot\frac{1}{\left(1+\frac{x}{ka}\right)^2}\right\}R_s(x)=0, \tag{7}$$

1) $\left.\frac{dR_s}{dx}\right|_{x=0} = -i\tilde{\delta}_g R_s(0)$, 2)* when $\operatorname{Im} k > 0$ $R_s(x) \in L^2[0, \infty)$.

Here $\tilde{\delta}_g$ — modified impedance, which depends, as is evident, from the value of the gradient of function $\varepsilon_m(x)$ on the surface of sphere, equal to

$$\tilde{\delta}_g = \sqrt{\varepsilon_m(0)}\delta_g - \frac{i}{2\varepsilon_m(0)}\cdot\left.\frac{d\varepsilon_m}{dx}\right|_{x=0}. \tag{8}$$

§2. Transition from operator L_r to the integral equation.

Analogous to [1] we consider the relationship/ratio

$$\frac{1}{\left(1+\frac{x}{ka}\right)^2} \simeq 1 - \frac{2x}{ka} \tag{9}$$

satisfied for all x. For the case of homogeneous medium the use of this relationship/ratio in (7) corresponds to transition from the cylindrical functions to their asymptotic representations through the Airy's functions. The possibility of this transition for the examined/considered case ka>>1 is proved in the work of V. A. Fok [12]. Introducing certain initial approximation/approach ν_s^0 to unknown eigenvalue ν_s and taking into account (9), let us write

equation (7) in the form

$$L_1 R_{\nu_s}(x) + L_2 R_{\nu_s}(x) = 0, \tag{10}$$

where

$$L_1 = \frac{d^2}{dx^2} + 1 + \delta \cdot \frac{\epsilon'_m(0)}{\epsilon_m(0)} \cdot x - \frac{\nu'_s(\nu'_s+1)}{(ka)^2}\left(1 - \frac{2x}{ka}\right), \tag{11}$$

$$L_2 = \frac{\epsilon_m(x)}{\epsilon_m(0)} - 1 - \delta \cdot \frac{\epsilon'_m(0)}{\epsilon_m(0)} \cdot x - \sqrt{\epsilon_m(x)}\, \frac{d^2}{dx^2}\, \frac{1}{\sqrt{\epsilon_m(x)}} + \frac{\nu'_s(\nu'_s+1) - \nu_s(\nu_s+1)}{(ka)^2}\left(1 - \frac{2x}{ka}\right). \tag{12}$$

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To differential expression (11) is related the linear part of function $\frac{\epsilon_m(x)}{\epsilon_m(0)}$, moreover before $\frac{\epsilon'_m(0)}{\epsilon_m(0)} x$ is set symbol $\delta = \begin{cases} 1 \\ 0 \end{cases}$. Symbol δ is introduced with that target in order not to write two different equations for the cases, when to L_1 one should carry $\frac{\epsilon'_m(0)}{\epsilon_m(0)} x$ and when this to make inexpediently. For the reduction of recording (11) (12) it is convenient to introduce the designations

$$\beta_s \equiv \frac{\nu_s(\nu_s+1)}{(ka)^2}, \quad \beta'_s \equiv \frac{\nu'_s(\nu'_s+1)}{(ka)^2},$$

$$\alpha_s^3 \equiv \left(\frac{2\beta'_s}{ka} + \delta \cdot \frac{\epsilon'_m(0)}{\epsilon_m(0)}\right)^{-\frac{2}{3}},$$

$$\eta_s \equiv \beta'_s - \beta_s, \quad \gamma_s \equiv \alpha_s^3(1 - \beta'_s), \quad \tilde{q}^3 \equiv i\,(\alpha_s^3)^{1/2}\,\tilde{\delta}_g$$

and the new variable

$$y \equiv (\alpha_s^3)^{-1/2} x + \gamma_s. \tag{13}$$

After this we turn operator L_1, by which we understand the operator, formed by differential expression (11) and boundary conditions 1)*, 2)*, and we obtain the integral equation

$$R_{\nu_s}(y) + \alpha_s^3 \eta_s \left[A R_{\nu_s}(t) + C R_{\nu_s}(t)\right] + B R_{\nu_s}(t) = 0. \tag{14}$$

Through A, C and B the integral operators

$$AR_{\nu_s}(y) \equiv \int_{\Gamma_s} K(y, t) R_{\nu_s}(t) dt, \tag{15}$$

$$CR_{\nu_s}(y) \equiv -\frac{2}{ka} (\alpha_s^0)^{1/2} \int_{\Gamma_s} K(y, t)(t-\gamma_s) R_{\nu_s}(t) dt, \tag{16}$$

$$BR_{\nu_s}(y) \equiv \alpha_s^0 \int_{\Gamma_s} K(y, t) \cdot \Phi(t) \cdot R_{\nu_s}(t) dt, \tag{17}$$

are designated. Furthermore, the nucleus

$$K(y, t) \equiv \begin{cases} u_1(y) \cdot \tilde{u}_1(t), & \gamma_s < t < y \\ \tilde{u}_1(y) \cdot u_1(t), & y \leqslant t, \end{cases} \tag{18}$$

$$\tilde{u}_1(y) \equiv \frac{1}{2i} [\sigma u_1(y) - u_2(y)], \tag{19}$$

$$\sigma \equiv \frac{u_2'(\gamma_s) + \tilde{q}^s u_2(\gamma_s)}{u_1'(\gamma_s) + \tilde{q}^s u_1(\gamma_s)}, \tag{20}$$

u_1 and u_2 - two linearly independent solutions of the equation

$$\frac{d^2u}{dy^2} + yu = 0, \tag{21}$$

moreover $u_1(y) \in L^2[\gamma_s, \infty]$, and $u_2(y) = -u_1(y)$.

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Function F(t), entering in (17), is determined by the equality

$$\Phi(t) = \frac{\epsilon_m(t)}{\epsilon_m(0)} - 1 - \tilde{\gamma} \cdot \frac{\epsilon_m'(0)}{\epsilon_m(0)} \cdot (\alpha_s^0)^{1/2} (t - \gamma_s) -$$
$$- \frac{1}{\alpha_s^0} \sqrt{\epsilon_m(t)} \frac{d^2}{dt^2} \frac{1}{\sqrt{\epsilon_m(t)}} \tag{22}$$

Instead of the duct/contour of integration Γ_s (Fig. 1) let us introduce the equivalent to it Γ_s' (Fig. 2), on which function $u_1(t)$

with $t \to \infty$ exponentially vanishes, providing the limitedness of operators A, C and B and, consequently, also the applicability of the Galerkin method for finding the eigenvalues $\alpha_s^2 \eta_s$ of equation (14).

§3. General/common/total expressions for the eigenvalues.

Without defining concretely thus far the form of the function $\varepsilon_m(x)$, we will obtain general/common/total expressions for eigenvalues ν_s in the first approximation. We search for ν_s in the form

$$\nu_s = ka\left(1 + \frac{t_s}{2\alpha_s^2}\right) \tag{23}$$

and in the same form we assign the initial approximations/approaches

$$\nu_s^l = ka\left(1 + \frac{t_s^l}{2\alpha_s^2}\right). \tag{24}$$

Subordinating α_s^2 to condition $\alpha_s^2 \gg 1$, we will have

$$\alpha_s^2 \eta_s \approx t_s^l - t_s. \tag{25}$$

Setting for itself as a goal to obtain sufficiently precise and simple analytical dependences t_s on the parameters of the problem, let us examine only two limiting cases - (I)-, when $|\tilde{q}_s| \ll 1$ and (II)-, when $|\tilde{q}_s| \gg 1$.

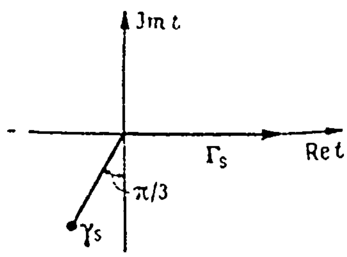

Fig. 1.

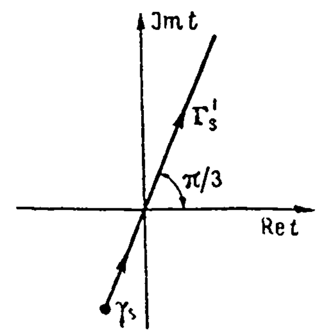

Fig. 2.

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Without repeating those given in [1] reasonings relative to the selection of initial approximations/approaches t_s^j (j=0) for the case (I), j=∞ for the case (II), let us recall their definition:

$$u_1'(-t_s^0)=0, \qquad (26)$$
$$(s=0,\ 1,\ 2,\ \ldots)$$
$$u_1(-t_s^\infty)=0. \qquad (27)$$

As the coordinate element/cell z_0 we select function $u^1(y)$, which satisfies condition (26) in the case (I) or (27) - in (II). Then in the first approximation, for the eigenvalues we obtain

$$t_s-t_s^j\simeq\frac{(u_1,\ \bar{u}_1)+(Bu_1,\ \bar{u}_1)}{(Au_1,\ \bar{u}_1)+(Cu_1,\ \bar{u}_1)}. \qquad (28)$$

Let us turn to formulas (18)-(20), that determines nucleus K(y, t). It is not difficult to note that σ is the high parameter both in the first and in the second case. Actually/really, when $|\tilde{q}'|\ll 1$ we have

$$\sigma\simeq\frac{u_2'(-t_s^0)}{u_1(-t_s^0)}\cdot\frac{1}{\tilde{q}'}, \qquad (29)$$

moreover the factor before $\frac{1}{\tilde{q}^{\bullet}}$ the order of one for several firsts t_s^0.

With $|\tilde{q}^{\bullet}| \gg 1$

$$\sigma \simeq \frac{u_2(-t_s^0)}{u_1'(-t_s^0)} \cdot \tilde{q}^{\bullet}, \tag{30}$$

consequently, and in this case $|\sigma|>>1$, since the factor before $\tilde{q}^{\bullet}$ the order of one. Taking into account this, K(y, t) is simplified, disregarding in it the members of order $1/\sigma$ in comparison with one. We will obtain

$$K(y,\ t) \simeq \frac{\sigma}{2i} u_1(y) \cdot u_1(t) \text{ with all } t \in \Gamma_s'. \tag{31}$$

Then formula (28) can be written thus:

$$t_s - t_s^l \simeq \frac{2i}{a_s \sigma}\left(1 + \frac{\sigma}{2i} b_s\right)\left(1 + \frac{c_s}{a_s}\right)^{-1}. \tag{32}$$

Here

$$a_s = \int\limits_{\Gamma_s'} u_1^2(t)\,dt = -\{\gamma_s u_1^2(\gamma_s) + u_1'^2 \gamma_s)\}. \tag{33}$$

$$c_s \equiv -\frac{2}{ka}(\alpha_s^0)^{1/2} \int\limits_{\Gamma_s'} (t - \gamma_s)\, u_1^2(t)\,dt = \frac{2}{ka}(\alpha_s^0)^{1/2} \frac{2}{3} \gamma_s a_s. \tag{34}$$

$$b_s \equiv \alpha_s^0 \int\limits_{\Gamma_s'} \Phi(t)\, u_1^2(t)\,dt \tag{35}$$

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From the designations

$$\beta_s^l = \frac{\nu_s^l(\nu_s^l + 1)}{(ka)^2},\ \nu_s^l = ka\left(1 + \frac{t_s^l}{2\alpha_s^0}\right)$$

introduced earlier it is evident that under condition $\alpha_s^3 \gg 1$, which is considered carried out, $\beta_s^j \simeq 1$. Taking into account this, expression for α_s^3 let us write in the form

$$\alpha_s^3 = \left(\frac{ka_э}{2}\right)^{2/3}, \quad (36)$$

where $a_э = \frac{a}{1+\delta\frac{\varepsilon_m'(0)}{\varepsilon_m(0)}\cdot\frac{ka}{2}}$ — equivalent radius, which is equal to a radius of sphere a for the homogeneous medium, when $\varepsilon_m'(0)=0$. In this case $\alpha_s^0 = \alpha_s = \left(\frac{ka}{2}\right)^{2/3}$. Relatively α_s we also assume that $\alpha_s \gg 1$. After returning to formulas (32)-(34), we see that

$$1+\frac{c_s}{a_s} = 1+\frac{1}{\alpha_s}\left(\frac{a_э}{a}\right)^{1/3}\frac{2}{3}\gamma_s. \quad (37)$$

Second term in (37) is negligibly small in comparison with one for several first numbers s by force $\alpha_s \gg 1$, therefore subsequently we will assume that

$$1+\frac{c_s}{a_s} = 1. \quad (38)$$

Formula (32) taking into account (29) (30) and (38) is converted to the form for (I) case

$$t_s - t_s^0 \simeq \frac{\tilde{q}^э}{t_s^0} + \frac{b_s}{a_s}\bigg|_{\gamma_s=-t_s^0}; \quad (39)$$

$$t_s - t_s^\infty \simeq \frac{1}{\tilde{q}^э} + \frac{b_s}{a_s}\bigg|_{\gamma_s=-t_s^\infty}. \quad (40)$$

During the conclusion/output of these relationships/ratios it is taken into consideration also that factors x_0 and γ_∞, which appear before first terms in (39) and (40), are in effect equal to one [1]. It is obvious, first term in (39) (40) corresponds to the equivalent "uniform" problem, in which instead of radius a and impedance $\tilde{\delta}_g$ figure equivalent radius $a_э$ and modified impedance $\tilde{\delta}_g$.

Component/term/addend $\frac{b_s}{a_s}$ in (39) (40), generally speaking, appears as a result of a difference in profile/airfoil $\varepsilon_m(x)$ from the linear; however, it is not equal to zero even for the linear profile/airfoil. The estimation of this component/term/addend with linear function $\varepsilon_m(x)$ is conducted in the following paragraph. At conclusion of this paragraph let us note that if we are interested in the wider range of change in parameter $\tilde{q}$, then instead of first term in (39) (40) it is possible to substitute the appropriate expressions from [2], obtained in the second approximation/approach of the method of moments/torques for the homogeneous medium. For this should be used formulas (8) and (9) from (2), having preliminarily replaced in them q by $\tilde{q}$.

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§4. Linear profile/airfoil.

For the linear profile/airfoil the proof of the concept of an equivalent radius usually is conducted at the level of differential equation (7). In this case the assumption about the possibility to disregard/neglect the so-called derivative of Schwarz $\sqrt{\varepsilon_m(x)}\frac{d^2}{dx^2}\frac{1}{\sqrt{\varepsilon_m(x)}}$. Substantially is utilized. It is considered also that upon transfer from the problem with the unhomogeneous atmosphere to the the equivalent to "uniform" the impedance of sphere does not change. In actuality in the equivalent "uniform" problem must figure the

modified impedance, which can considerably differ from δ_g by very low frequencies. It is of interest to rate/estimate the effect of the named factors on eigenvalues t_s. Since for both cases (I) and (II) in question in the distant zone the field is determined by one normal wave, to which eigenvalue t_0 corresponds, we will obtain analytical expressions only for this eigenvalue. For the same reason and for the exponential profile/airfoil we will be interested only in zero eigenvalue. Considering that

$$\varepsilon_m(x) = \varepsilon_m(0) + \varepsilon_m'(0)\,x, \qquad (41)$$

entire function $\varepsilon_m(x)$ is related to operator L_1 (11), for which we assume/set δ=1. In this case we have

$$a_9 = \frac{a}{1 + \frac{\varepsilon_m'(0)}{\varepsilon_m(0)} \cdot \frac{ka}{2}}. \qquad (42)$$

For future reference it is convenient to introduce parameter η_9 according to the formula

$$\eta_9 \equiv -\frac{\varepsilon_m'(0)}{\varepsilon_m(0)} \cdot \left(a_9^9\right)^{1/2}. \qquad (43)$$

We will consider it its small in comparison with one: $|\eta_9| \ll 1$. Under the conditions, characteristic for the earth's atmosphere, this parameter, for example, has a value $\eta_9 = 0.0063$ at the frequency f=5 kHz and with an increase in the frequency decreases, vanishing with f→∞. Let us examine now coefficient of b_0, which determines second term in (39) (40) with s=0

$$b_0 = \int_{\gamma_1}^{\infty e^{i\pi/3}} \Phi(t)\, u_1^2(\ )\, dt \qquad (44)$$

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Here

$$\Phi(t) = -\frac{3}{4}\eta_э^2 \frac{1}{[1 - \varepsilon_э(t - \gamma_0)]^2}. \qquad (45)$$

Let us recall that $\gamma_0 \approx -t_0^j (j=0, \ldots)$. To express integral (44) through the known functions is impossible, but it is possible to calculate approximately it. For this we will use smallness $\eta_э$ and fact that the basic contribution to the integral gives region small t. This follows from the character of the behavior of function $u_1(t)$, qualitatively depicted in Fig. 3, where λ - certain complex coefficient. Decomposing/expanding F(t) in the series/row according to degrees $\eta_э$ it is not difficult to obtain correction to eigenvalue t_0 due to Schwarz's derivative. We will be restricted to the dominant relatively $\eta_э$ term

$$\frac{b_0}{a_0} = -\frac{3}{4}\eta_э^2 [1 + o(\eta_э)]. \qquad (46)$$

Let us turn now to the expression for $\tilde{q}^э$. Taking into account (8) and (43), let us write it in the form

$$\tilde{q}^э = q^э + \frac{1}{2}\eta_э, \qquad (47)$$

where

$$q^э = i\left(\frac{ka_э}{2}\right)^{1/3} \sqrt{\varepsilon_m(0)}\,\delta_g \qquad (48)$$

As is evident, q' in the accuracy corresponds to the concept of an equivalent radius, since it is obtained from q for the homogeneous medium, replacement of radius a on $a_э$. Component/term/addend $\frac{1}{2}\gamma_{0}$, in (47) is correction to the concept of an equivalent radius because in the presence of inhomogeneous medium instead of impedance δ_g appears modified impedance $\tilde{\delta}_g$.

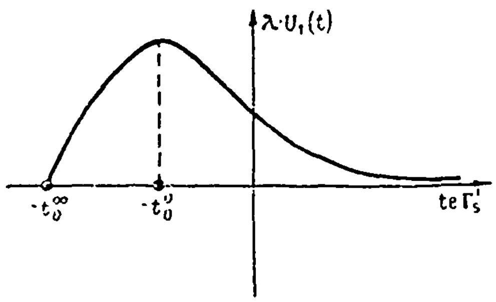

Fig. 3.

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It is clear that this component/term/addend can prove to be essential only in small ones $|q^{э}|$ We will obtain the resultant expressions for t_0, after substituting (46) (47) in (39) (40) with s=0

$$\text{(I)}\ t_0 - t_0^0 \simeq \frac{q^{э}}{t_0^0} + \frac{1}{2t_0^0}\eta_{э} - \frac{3}{4}\eta_{э}^2\left[1 + o(\eta_{э})\right], \qquad (49)$$

$$\text{(II)}\ t_0 - t_0^{\infty} \simeq \frac{1}{q^{э}} - 3\ \eta_{э}^2\left[1 + o(\eta_{э})\right]. \qquad (50)$$

Thus, even with the linear profile/airfoil the concept of an equivalent radius can be considered justified only in such a case, when in (49) (50) the terms, which contain parameter $\eta_{э}$ are sufficiently small. Let us rate/estimate their effect on the phase speed and attenuation of zero normal wave. For this let us examine the linear profile/airfoil, characteristic for the lower layers of the earth's atmosphere

$$\varepsilon_m(x) = 1 + \alpha - \frac{\alpha}{k_0 H} x \qquad (51)$$

with $\alpha=6.4\cdot10^{-4}$, H=7.4 km.

Latter/last component/term/addend in (49) (50), that appears due to Schwarz's derivative, obviously, does not affect fading; it it is possible not to consider, also, during the calculation of phase speed, since it the second order of smallness in comparison with the contribution due to the modified impedance. For the certainty we assume/set $\delta_K=0$. Table 1 gives values $\frac{v^{(0)}_{\text{phs}}}{c}$ and $\alpha^{(0)}/k_0$ without the account and taking into account second term in (49) for several frequencies.

Table 1.

(1) f, кгц	$v^{(0)}_{\phi}$, с (2)		$a^{(0)}/k_0$	
	а	б	а	б
0,5	0,4302	0,4268	2,29	2,261
5,0	0,2596	0,2589	4,942	4,927
50	0,1399	0,1398	10,617	10,640

Note: a) without the account; b) taking into account second term in (49) for several frequencies.

Key: (1). kHz. (2). s.

§5. Exponential profile/airfoil.

For the exponential profile/airfoil we will examine the cases of high and low frequencies separately. The reason for this separation consists in the fact that eigenvalues λ for these cases substantially differ from each other. It proves to be, and this it was possible to assume, on the basis of the physical considerations that at the high frequencies the eigenvalues for the exponential profile/airfoil were close to the appropriate eigenvalues for the linear profile/airfoil, and on the low frequencies they were located near the eigenvalues of problem with the homogeneous medium.

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Therefore it is necessary to introduce different initial approximations/approaches ν for the high and low frequencies in order already in the first approximation, of the Galerkin method to obtain sufficiently precise formulas for unknown eigenvalues ν. As already mentioned in the preceding/previous paragraph, we will be restricted to the investigation of zero eigenvalue ν_0, moreover let us disregard/neglect Schwarz's derivative. Based on the example of linear profile/airfoil it was shown that its contribution to the eigenvalue of second order of smallness in comparison with the contribution of the modified impedance.

High frequencies. For the high frequencies the linear part of function $\varepsilon_m(x)$, given by the formula

$$\varepsilon_m(x) = 1 + \alpha \cdot e^{-\frac{x}{kH}}, \tag{52}$$

it is carried to operator L_1(11), assuming/setting $\delta=1$. Coefficient b_0 in this case determines correction to eigenvalue t_0 due to a difference in the exponential profile/airfoil from the linear and takes the form

$$b_0 = \frac{\alpha}{1+\alpha}\int_{t_0}^{\infty} \{e^{-\mu_9(t-t_0)} - 1 + \mu_9(t-t_0)\}\, u_1^2(t)\,dt, \tag{53}$$

where

$$\mu_9 = \frac{(\alpha_9)^{1/2}}{kH}, \quad \alpha_9 = \left(\frac{ka_9}{2}\right)^{2/3}.$$

Equivalent radius $a_э$ is determined by formula (42). We will consider high those frequencies, with which is fulfilled the inequality

$$\mu_э \ll 1.$$

In order to represent the real values of parameter $\mu_э$ let us note that under the conditions of the earth's atmosphere, when $\alpha \simeq 6.4 \cdot 10^{-4}$, $H \simeq 7.4$ km, we have $\mu_э \simeq 0{,}06$ at frequency f=10 MHz. With an increase in frequency $\mu_э$ it decreases, vanishing with $f \to \infty$. Three times integrating in parts first term in (53) and utilizing smallness of parameter $\mu_э$, it is not difficult to obtain the approximation

$$b_0 = -\frac{\alpha}{1+\alpha} \cdot \frac{a_э}{2H} \mu_э \left\{ \frac{4}{35} \left[\gamma_0^3 u_1^2(\gamma_0) + \gamma_0^2 u_1'^2(\gamma_0) \right] + \frac{1}{10} u_1^2(\gamma_0) \right\} \times \\ \times [1 + o(\mu_э)]. \quad (54)$$

For conclusion/output (54) recursion relations (П8)-(П10) of work [1] were used, and it is taken into consideration, that $u_1(\gamma_0) \cdot u'_2(\gamma_0) = 0$ in view of the selection of initial approximation/approach. It is obvious, with $\omega \to \infty$ coefficient $b_0 \to 0$, therefore, with $\omega \to \infty$ disappears the difference between the exponential and linear profile/airfoil. This bears out the fact that only the lower layers of the atmosphere, where the profile/airfoil is close to the linear, play main role in shaping of the distant field of high frequency.

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Substituting (54) in (39) (40) and taking into account (47) (33), we come to the final formulas for eigenvalue t_0 in the case of the high

frequencies

$$\text{(I)}\quad t_0 - t_0^0 \simeq \frac{q^э}{t_0^0} + \frac{1}{2t_0^0}\eta_э + \frac{\alpha}{1+\alpha}\frac{a_э}{2H}\frac{\frac{4}{3\cdot 5}(t_0^0)^3 - \frac{1}{10}}{t_0^0}\mu_э[1 + 0(\mu_э)], \quad (55)$$

$$\text{(II)}\quad t_0 - t_0^\infty \simeq \frac{1}{q^э} + \frac{\alpha}{1+\alpha}\cdot\frac{a_э}{2H}\cdot\frac{4}{3\cdot 5}(t_0^\infty)^2\mu_э[1 + 0(\mu_э)]. \quad (56)$$

Table 2 gives formulas (55) obtained with the use of the value of phase speed and attenuation of zero normal wave when $\delta_g=0$ for the linear and exponential profiles/airfoils.

Low frequencies. Assuming that at the low frequencies the role of the atmospheric heterogeneity of exponential profile/airfoil is not so/such essential as on the high ones, we will search for eigenvalues in the vicinity of the eigenvalues of problem with the homogeneous medium, in which $k=k_0$. For this of the function $v_m(x)$ to operator L_1 (11) one is carried, assuming/setting $\delta=0$, and by k everywhere we understand k_0. Then coefficient b_0, which determines the effect of tropospheric refraction, is written as follows:

$$b_0 = \alpha\sigma_s \int_{\gamma_0}^{\infty e^{i\pi/3}} e^{-\mu(t-\gamma_0)} u_1^2(t)\,dt. \quad (57)$$

Here

$$\mu \equiv \frac{\alpha_s^{1/2}}{k_0 H}, \quad \sigma_s = \left(\frac{k_0 a}{2}\right)^{2/3}.$$

Low we will call those frequencies, with which is fulfilled the inequality

$$\mu \gg 1. \quad (58)$$

For the earth's atmosphere at the frequency f=5 kHz we have μ=8.94. With the decrease of frequency the parameter μ increases. Three times integrating in parts (57) and considering as that carried out relationship/ratio (58), easily we come to the approximation

$$b_0 = \alpha\varkappa_s \frac{\mu^3}{2+\mu^3}\left\{\frac{u_1^2(\gamma_0)}{\mu} + 2\,\frac{\gamma_0 u_1^2(\gamma_0) + u_1'^2(\gamma_0)}{\mu^3} + o\left(\frac{1}{\mu^4}\right)\right\}. \quad (59)$$

Table 2.

(1) f, Мгц	$v^{(0)}_{фаз}/c$ (2)		$\alpha^{(0)}/k$	
	a	б	a	б
20	0,021602	0,021741	78,41	78,95
50	0,016008	0,016064	106,47	106,84

Note: a) for the linear profile/airfoil; b) for the exponential profile/airfoil.

Key: (1). MHz. (2). s.

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It is obvious, with the decrease of the frequency, when $\mu\to\infty$, coefficient $b_0\to 0$, and heterogeneity of the atmosphere in this case becomes apparent only through the modified impedance. After substitution (59) in (39) (40) we obtain the approximation analytical formulas for t_0, valid at the low frequencies

$$\text{(I)}\quad t_0 - t_0^0 \simeq \frac{q}{t_0^0} + \frac{1}{2t_0^0}\eta + \alpha\alpha_s \frac{2}{2+\mu^3}\left(1+\frac{\mu^2}{2t_0^0}\right)\left[1+o\left(\frac{1}{\mu}\right)\right], \qquad (60)$$

$$\text{(II)}\quad t_0 - t_0^\infty \simeq \frac{1}{q} - \alpha\cdot\alpha_s\cdot\frac{2}{2+\mu^3}\left[1+o\left(\frac{1}{\mu}\right)\right]. \qquad (61)$$

During writing (60) it is taken into consideration, that in this case $\tilde{q}^9 = q + \frac{1}{2}\eta$, since $a_s = a_0$. In formulas (60) (61) term $o(1/\mu)$ is small in comparison with one not only because $\mu>>1$, but also due to the character of the behavior of integrand in the rejected integral. This

provides applicability (60) (61) in the wider frequency region, when $\mu>1$.

§6. Solution of inverse problem.

In our work [2] inverse problem for the impedance sphere, which is located in the homogeneous medium, was solved by amplitude method. The use/application of this method for solving the inverse problem taking into account the atmospheric heterogeneity of exponential profile/airfoil composes the content of present paragraph. We consider source and receiver as those arranged/located on the surface of sphere and as in [2], we will solve problem under the assumption of single-modality of the propagation, when

$$E_r \approx \frac{i\omega\mu_0 I h_g}{4a}\sqrt{\frac{2}{\pi k a\theta}}\sqrt{\frac{\theta}{\sin\theta}}\, e^{i\pi/4} e^{ika\theta}\Lambda_0 e^{ixt_0}. \qquad (62)$$

Here x - given distance from the source to observation point

$$x = \frac{ka}{2}\cdot\frac{1}{a_э^2}\theta = \left(\frac{a}{a_э}\right)^{2/3}\left(\frac{ka}{2}\right)^{1/3}\theta. \qquad (63)$$

Taking the ratio of amplitudes $|E_r(x_1)|$ and $|E_r(x_2)|$, measured at frequencies ω_i at two diverse points x_1 and x_2, we will obtain system of equations for the determination of the electrical parameters, which determine impedance, and the parameters of the heterogeneity of the atmosphere

$$\left(\frac{a}{a_э}\right)^{2/3}\operatorname{Im} t_0\,[p_k,\ \omega_i] = \Phi_{12}(\omega_i) \quad (i,\ k=1,\ 2,\ \ldots,\ N), \qquad (64)$$

where

$$\Phi_{12}(\omega_i) = \frac{1}{\left(\frac{ka}{2}\right)^{1/3}(\theta_2-\theta_1)} \ln \sqrt{\frac{\sin\theta_1}{\sin\theta_2}} \left|\frac{E_r(x_1)}{E_r(x_2)}\right|, \qquad (65)$$

and p_k — unknown parameters, whose number is equal to N.

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With known analytical expressions $t_0[p_k, \omega_i]$ the solution of system (64) will not compose labor/work and can be obtained, for example, by Newton- Kantorovich method [13]. The necessary expressions for t_0 are obtained in the preceding/previous paragraph in two limiting cases (I) and (II) both on the high ones and at the low frequencies. If necessary it is possible to utilize also formulas (8) (9) work [2], as already mentioned earlier.

Here we will pause at the case, when system (64) admits analytical solution relative to parameters a and H. let us examine the frequency band, bounded above by condition $\mu \gg 1$, and from below by the frequencies, at which still it is possible to assume/set $\tilde{q}^* \simeq q$, so that

$$t_0 - t_0^0 \simeq \frac{q}{t_0^0} + a\alpha_s \frac{2}{2+\mu^1}\left(1 + \frac{\mu^2}{2t_0^0}\right). \qquad (66)$$

In the lower part of this range with μ>>1 the effect of tropospheric refraction is negligibly small; therefore in this part of the

spectrum it is possible to determine the impedance of sphere δ_g, counting the medium of uniform. This problem is solved in [2]; therefore we will consider impedance as the function of frequency known. However, for determining α and H we will use the higher frequencies of the range in question, when the effect of tropospheric refraction is noticeable. Taking into account (66) from (64) we obtain system of two equations relatively α and H

$$\alpha \frac{H\gamma^2(\omega_i)}{2H^3+\gamma^3(\omega_i)} = \varphi(\omega_i) \quad (i=1,\ 2). \tag{67}$$

Function $\varphi(\omega_i)$ is determined by the equality

$$\varphi(\omega_i) = -\frac{|t_0^0|}{\sigma_s \sin \pi/3}\left\{\Phi_{12}(\omega_i) + \operatorname{Im}\left(t_0^0 + \frac{q}{t_0^0}\right)\right\} \tag{68}$$

and it is considered known. Parameter $\gamma(\omega) \equiv \frac{\sigma_s^{1/2}}{k_0} = \frac{1}{k_0}\left(\frac{k_0 a}{2}\right)^{1/3}$. The solution of system (67) is located elementarily, and we immediately extract it

$$H = \frac{\gamma_1}{2^{1/3}} \frac{\left\{\frac{\varphi_1}{\varphi_2} - \frac{\gamma_2}{\gamma_1}\right\}^{1/3}}{\left\{\frac{\gamma_1^2}{\gamma_2^2} - \frac{\varphi_1}{\varphi_2}\right\}^{1/3}}, \tag{69}$$

$$\alpha = \frac{2H^3+\gamma_i^3}{H\gamma_i^2}\varphi_i. \tag{70}$$

For the reduction of recording the designations

$$\gamma(\omega_i) \equiv \gamma_i, \quad \varphi(\omega_i) \equiv \varphi_i. \tag{71}$$

are here introduced. In formula (70) the selection of mark i, naturally, does not play role.

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REFINEMENT OF BOOKER'S EQUATION FOR CYLINDRICAL LAYERED INHOMOGENEOUS ANISOTROPIC MEDIA.

V. M. Vyatkin.

The known equation of Booker [1] is obtained for the case of plane-layered anisotropic medium. But the real ionosphere possesses final curvature and terrestrial magnetic field is heterogeneous. Therefore it is of interest to refine Booker's equation taking into account these two effects. We selected the cylindrical model of the ionosphere, which makes it possible in the first approximation, to consider the curvature of the real ionosphere.

Let us introduce cylindrical coordinate system (ρ, φ, z) and we will consider the ionosphere as layered inhomogeneous on ρ the anisotropic medium, which occupies the region of space with $\rho \gg b$. We consider subsequently earth's magnetic field heterogeneous, which depend only on ρ (heterogeneity of field in the directions, the tangents to the earth's surface, as are shown calculations, can be disregarded/neglected).

The general/common/total equation

$$\mathrm{rot}\,\mathrm{rot}\,\mathbf{E}-k_0^2\dot{\varepsilon}_m\mathbf{E}=0, \qquad (1)$$

where $k_0=\omega\sqrt{\epsilon_0\mu_0}$ - the wave number of vacuum, is initial in the study of the problem about the propagation of electromagnetic waves in the anisotropic media, $\dot{\varepsilon}_m$ — tensor of the relative complex dielectric constant of medium.

The solution of equation (1) we will search for in the form of series/row according to the reverse/inverse degrees of k_0. Substituting this series/row in equation (1) and equalizing the coefficients, which stand in the equal degrees of k_0, we obtain the chain/network of the engaged equations, the first of which takes the form

$$[\nabla\Psi[\nabla\Psi\cdot\mathbf{E}^{(0)}]]+\dot{\varepsilon}_m\mathbf{E}^{(0)}=0. \qquad (2)$$

It is not difficult to show that in the case of cylindrical layered inhomogeneous (on ρ) anisotropic medium interesting us the vector $\Delta\Psi$ has components

$$\nabla\Psi=\left(\frac{\partial\Psi}{\partial\rho};\ \frac{\rho_0}{\rho}C_1;\ C_2\right), \qquad (3)$$

where $C_1=\cos\gamma_0$ and $C_2=\cos\alpha$, and α has the sense of the angle between the direction of wave and z axis; γ_0 can be considered as the angle, formed by zonal harmonic [2] (by spiral wave) of order ν with the direction, by tangent to the cylinder of radius $k_0\rho_0$(cos

$\gamma_0=\nu/k_0\rho_0$); ρ_0 - coordinate of the point of the position of the source, which we will assume/set by that arranged/located lower than the boundary of ionosphere (ρ_0<b).

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For value $\partial\Psi/\partial\rho=q(\rho)$ from the condition for existence of the nontrivial solution of equation (2) the equation, which is the analog of Booker's equation [1] in the cylindrical case

$$F(q)=\alpha q^4+\beta q^3+\gamma q^2+\delta q+\epsilon=0. \qquad (4)$$

is obtained.

Let us note that equation (4) is obtained from "Cartesian" biquadraticof Booker [1] by replacement of C_1 by $(\rho_0/\rho)C_1$, which follows from comparison (3) with the expression for components $\nabla\Psi$ in the flat/plane case. Therefore expressions for the coefficients α, β, γ, δ, ϵ of equation (4) take the same form, as in flat/plane case [1] with replacement of C_1 by $(\rho_0/\rho)C_1$.

If we in the formulas for the coefficients of equation (4) assume $\frac{\rho_0}{\rho}\simeq\frac{a}{a+\zeta}\simeq 1-\eta$, where $\eta=\zeta/a$ (a - radius of the earth)[1], and to consider that terrestrial magnetic field in the first approximation, it carries dipole character, so that at the height/altitude ζ above the earth's surface

$$H^{(0)}(a+\zeta)=\frac{H_a^{(0)}a^3}{(a+\zeta)^3}\simeq H_a^{(0)}(1-3\eta), \qquad (5)$$

where $\eta=\zeta/a$; $H_a^{(0)}$ — field on the earth's surface, then expression for the coefficients α, β, γ, δ, ϵ with an accuracy to the values of order η they will take the form

$$\alpha=U(U^2-Y_a^2)-X(U^2-l^2Y_a^2)+6\eta Y_a^2(U-lX), \qquad (6)$$

$$\beta=2XlY_a^2\{(C_1m+C_2n)-\eta(7C_1m+6C_2n)\}, \qquad (7)$$

$$\begin{aligned}\gamma=&-S^2[2U^2(U-X)-2UY_a^2+XlY_a^2]+2XU(U-X)+\\&+XY_a^2[-1+(C_1m+C_2n)^2]-\\&-2\eta C_1^2[2U^2(U-X)-2UY_a^2+XlY_a^2]-6\eta S^2Y_a^2(2U-Xl^3)-\\&-2\eta XY_a^2\{C_1m(C_1m+C_2n)+3[-1+(C_1m+C_2n)^2]\}, \qquad (8)\end{aligned}$$

$$\delta=-2XlY_a^2\{(S^2+2\eta C_1^2)(C_1m+C_2n)-\eta S^2(7C_1m+6C_2n)\}, \qquad (9)$$

$$\begin{aligned}\epsilon=&(U-X)[U^2(S^4+4\eta C_1S^2)-2XU(S^2+2\eta C_1^2)+X^2]+\\&+S^2\{XY_a^2\{1-(C_1m+C_2n)^2+2\eta C_1m(C_1m+C_2n)-\\&-6\eta[1-(C_1m+C_2n)^2]\}-UY_a^2(S^2+2\eta C_1^2-6\eta S^2)\}+\\&+2\eta C_1^2Y_a^2\{X[1-(C_1m+C_2n)^2]-US^2\}, \qquad (10)\end{aligned}$$

where

$$S_2^2=1-C_2^2;\quad C^2=C_1^2+C_2^2;\quad S^2=1-C^2,\quad Y_a^2=\left(\frac{\omega_{на}}{\omega}\right)^2,$$

$\omega_{на}$ — the value of gyrofrequency on the earth's surface $X=\frac{\omega_{пл}^2}{\omega^2}$, $\omega_{пл}$ — plasma frequency; $U=1+i\frac{\nu_{эфф}}{\omega}$, $\nu_{эфф}$ — effective collision frequency; l, m, n - direction cosines of vector with the axes of coordinates ρ, φ, z.

FOOTNOTE [1]. Let us note that in work [3] the coefficient η, which considers the final curvature of the ionosphere, derived from the geometric examination, is obtained erroneously ($2\zeta/a$ instead of ζ/a).

ENDFOOTNOTE.

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From the examination of equation (4) it is possible to obtain the information abou' the bias/displacement of the roots of Booker's equation [1], caused by the presence of curvature in the ionosphere and by the heterogeneity of terrestrial magnetic field. Any simple analytical expressions for this it is impossible to obtain, but qualitatively it is possible to say that the values of the roots of equation (4) decrease in comparison with the flat/plane case, but the effect of the final curvature of ionosphere and magnetic bump of the earth/ground on the height/altitude noticeably is manifested in the region, where approximation/approach WKB becomes inapplicable.

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